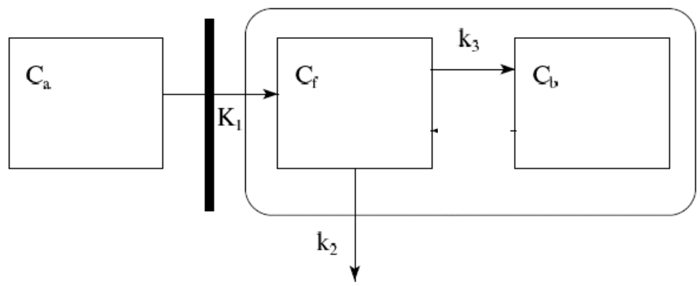
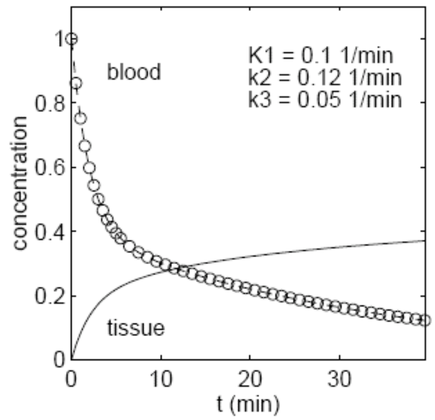
2 tissue-compartment *irreversible* model

Energy metabolism (FDG) - two tissue compartment model

2 tissue-compartment model

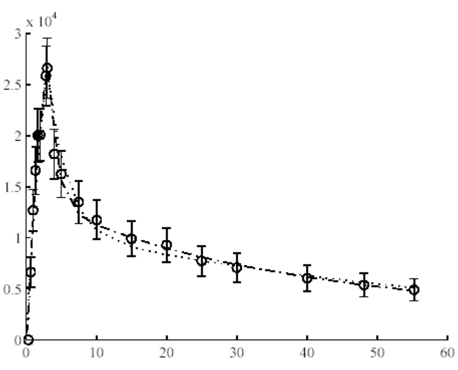


Tissue uptake



Maguire PET PK Course 2008

Input function



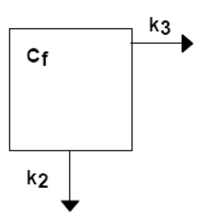
Maguire PET PK Course 2008

Differential equations

$$\frac{dc_f}{dt} = K_1 c_a - (k_2 + k_3)c_f$$

$$\frac{dc_b}{dt} = k_3 c_f$$

Simplification



washed out $\frac{k_2}{(k_2 + k_3)}$ and metabolised $\frac{k_3}{(k_2 + k_3)}$.

c_f after a 'delta' input:

$c_f = K_1 e^{-(k_2+k_3)t}$

$$c_b = k_3 \int_0^t c_f(\tau) d\tau = k_3 \int_0^t K_1 e^{-(k_2+k_3\tau)} d\tau = \frac{K_1 k_3}{(k_2 + k_3)}\left(1 - e^{-(k_2+k_3)t}\right)$$

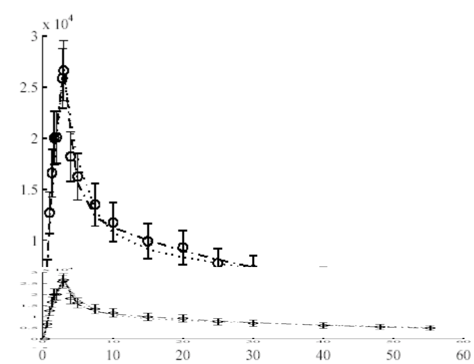
2-tissue irreversible solution

$$c_b = k_3 \int_0^t c_f(\tau) d\tau = k_3 \int_0^t K_1 e^{-(k_2 + k_3 \tau)} d\tau = \frac{K_1 k_3}{(k_2 + k_3)} \left(1 - e^{-(k_2 + k_3)t}\right)$$

$$K_1 \cdot \left(\frac{k_2}{(k_2 + k_3)} e^{-(k_2+k_3)t} + \frac{k_3}{(k_2 + k_3)} \right) = \text{e-function + const}$$

$$K_1 \left(e^{-(k_2+k_3)} + \frac{k_3}{(k_2 + k_3)} \left(1 - e^{-(k_2+k_3)t}\right) \right) = \text{free+ trapped} = c_f + c_b$$

Intravascular



$+V_b c_p$

System response: Constant + exponential

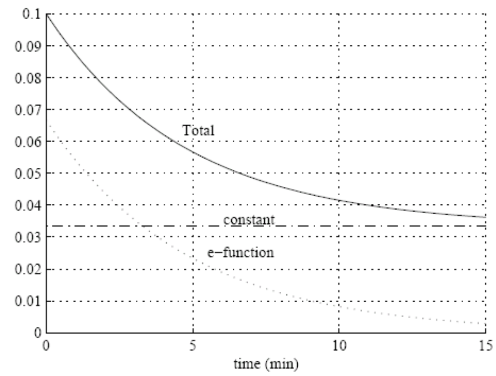


Figure 5-3: Sketch of equation (5.7) for [K_1,k_2,k_3]=[0.1,0.14,0.07] min^{-1}.

System Response: Free + Trapped

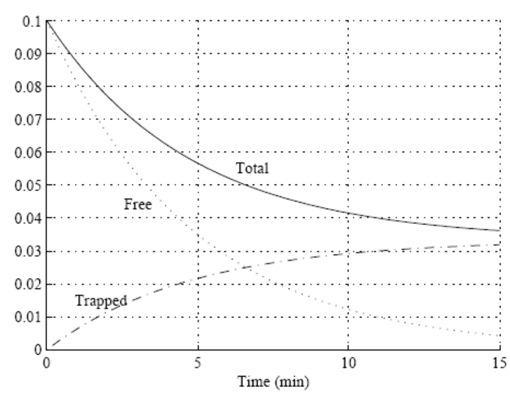


Figure 5-4: Sketch of equation (5.8) for [K_1,k_2,k_3]=[0.1,0.14,0.07] min^{-1}.

Macro parameters

$$K_i = \frac{K_1 k_3}{k_2 + k_3}$$

$$CMR_{glu} = K_i \left(\frac{c_p^g}{LC} \right)$$

$$SUV = \frac{C_{PET} \; W}{A}$$